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Discussion

Cooperation and wealth[☆]

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Abstract

We calculate the equilibrium fraction of cooperators in a population in which payoffs accrue from playing a single-shot prisoner's dilemma game. Individuals who are hardwired as cooperators or defectors are randomly matched into pairs, and cooperators are able to perfectly find out the type of a partner to a game by incurring a recognition cost. We show that the equilibrium fraction of cooperators relates negatively to the population's level of wealth.

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1. Introduction

An example illustrates that the level of wealth of a population and the equilibrium fraction of cooperators in a population are inversely related. It has been argued that the fraction of cooperators in a large society can be expected to be smaller than the fraction of cooperators in a small society (Binmore, 1998; Cook and Hardin, 2001). To the extent that a large society (say a city) is wealthier than a small society (say a town), the size effect may conceal a wealth effect.

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2. The game and the payoffs

Consider the following two-player, two-strategy game in which a player who cooperates gets a payoff of R if his opponent cooperates, and S if the opponent defects. A player who defects gets T if his opponent cooperates, and P if the opponent defects. The game is a prisoner's dilemma game: $T > R > P > S$. Hence defection is the dominant strategy for each player.

Let there be a large population of players consisting of individuals who are hardwired to be cooperators and individuals who are hardwired to be defectors. Individuals are randomly matched into pairs. An individual does not know the type of the individual with whom he is matched, but he can obtain such information at a cost, $0 < K < \bar{K}$, where $\bar{K}$ will be defined below. The type-recognition test is perfect. Thus, if an individual chooses to incur the cost and administer the test, the individual finds out whether he is matched with a cooperator or with a defector. The individual can then decide to play or not to play. If the individual decides not to play, he randomly picks another individual from the population and administers the type-recognition test in the new match. If individuals agree to play, they play their hardwired strategies, receive their respective payoffs, and leave the partner-seeking population to be replaced by new individuals. In equilibrium (to be characterized below) the flow of individuals of each type who enter the population exactly replaces the flow of individuals of each type who exit the population.

3. The types and their expected payoffs

Following Stark (1999, chapter 5), we study a population that consists of three types: defectors who play without incurring a recognition cost, cooperators who play after incurring the recognition cost, and cooperators who play without incurring the recognition cost. While there can be an equilibrium with all three types present and an equilibrium with defectors only, (i) there cannot be an equilibrium without defectors; and (ii) there cannot be an equilibrium with only defectors and non-testing cooperators. The rationale for (i) is that there cannot be an equilibrium with only non-testing cooperators because defectors will do better than cooperators; there cannot be an equilibrium with only testing cooperators because non-testing cooperators will do better; and there cannot be an equilibrium with only both types of cooperators because the non-testing cooperators will do better than the testing cooperators. The rationale for (ii) is that there cannot be an equilibrium with only defectors and non-testing cooperators because defectors will do better than the non-testing cooperators.

Let the steady-state fractions of testing cooperators, non-testing cooperators, and defectors be π_{t}, π_{nt}, and π_{d}, respectively, $\pi_{\mathrm{t}} + \pi_{\mathrm{nt}} + \pi_{\mathrm{d}} = 1$. Given the manner in which a testing cooperator acts and plays, his expected payoff is

$$V_{\mathrm{t}} = R - \frac{K}{1 - \pi_{\mathrm{d}}}. \tag{1}$$

The proof is as follows: the expected net payoff from administering the cost K (exactly once) and encountering a cooperator in the first match is $R(1 - \pi_{\mathrm{d}}) - K(1 - \pi_{\mathrm{d}})$; from failing to encounter a cooperator in the first match but encountering one in the second match is $R\pi_{\mathrm{d}}(1-\pi_{\mathrm{d}})-2K\pi_{\mathrm{d}}(1-\pi_{\mathrm{d}})$; from failing to encounter a cooperator in the first two matches but succeeding in encountering one in the third match is $R\pi_{\mathrm{d}}^2(1-\pi_{\mathrm{d}})-3K\pi_{\mathrm{d}}^2(1-\pi_{\mathrm{d}})$; and so on. Thus,

$$\begin{aligned}
V_{\mathrm{t}} &= R(1-\pi_{\mathrm{d}}) - K(1-\pi_{\mathrm{d}}) + R\pi_{\mathrm{d}}(1-\pi_{\mathrm{d}}) - 2K\pi_{\mathrm{d}}(1-\pi_{\mathrm{d}}) \\
&\quad + R\pi_{\mathrm{d}}^2(1-\pi_{\mathrm{d}}) - 3K\pi_{\mathrm{d}}^2(1-\pi_{\mathrm{d}}) + \cdots \\
&= \frac{R(1-\pi_{\mathrm{d}})}{1-\pi_{\mathrm{d}}} - K(1-\pi_{\mathrm{d}})(1+2\pi_{\mathrm{d}}+3\pi_{\mathrm{d}}^2+\cdots) \\
&= R - K(1-\pi_{\mathrm{d}})[(1+\pi_{\mathrm{d}}+\pi_{\mathrm{d}}^2+\cdots)+(\pi_{\mathrm{d}}+\pi_{\mathrm{d}}^2+\cdots)+(\pi_{\mathrm{d}}^2+\cdots)+\cdots] \\
&= R - K(1-\pi_{\mathrm{d}})\left(\frac{1}{1-\pi_{\mathrm{d}}}+\frac{\pi_{\mathrm{d}}}{1-\pi_{\mathrm{d}}}+\frac{\pi_{\mathrm{d}}^2}{1-\pi_{\mathrm{d}}}+\cdots\right) \\
&= R - K(1-\pi_{\mathrm{d}})\frac{1/(1-\pi_{\mathrm{d}})}{1-\pi_{\mathrm{d}}} = R - \frac{K}{1-\pi_{\mathrm{d}}}. \qquad \square
\end{aligned}$$

The expected payoff of a non-testing cooperator who plays the game with whomever he is paired with in the first match is

$$V_{\mathrm{nt}} = (1-\pi_{\mathrm{d}})R + \pi_{\mathrm{d}}S. \tag{2}$$

Since a defector always plays, that is, he plays when matched either with a non-testing cooperator or with a defector, his expected payoff is

$$V_{\mathrm{d}} = \frac{1-\pi_{\mathrm{t}}-\pi_{\mathrm{d}}}{1-\pi_{\mathrm{t}}}T + \frac{\pi_{\mathrm{d}}}{1-\pi_{\mathrm{t}}}P = T - \frac{\pi_{\mathrm{d}}}{1-\pi_{\mathrm{t}}}(T-P). \tag{3}$$

4. Equilibrium with defectors and testing cooperators but without non-testing cooperators

From the discussion in the preceding section it follows that an equilibrium with defectors and testing cooperators but without non-testing cooperators is feasible. If there are no non-testing cooperators, $\pi_{\mathrm{t}}+\pi_{\mathrm{d}}=1$; the expected payoff of testing cooperators is $V_{\mathrm{t}} = R-(K/\pi_{\mathrm{t}})$; and the expected payoff of defectors (who can play only with defectors) is $V_{\mathrm{d}} = P$. In equilibrium, testing cooperators receive the same expected payoff as defectors. Thus,

$$R - \frac{K}{\pi_{\mathrm{t}}} = P$$

or

$$\pi_{\mathrm{t}} = \frac{K}{R-P}, \tag{4}$$

assuming that $K < R-P \equiv \bar{K}$.

To help unravel the nature of the equilibrium, consider alternative values of K. Suppose that K were equal to $R-P$. π_{t} would then be equal to one. But having a population with only testing cooperators cannot be an equilibrium because in that case the non-testing cooperators will do better. Thus, we have a contradiction. Suppose that $K \to 0$. It follows that $\pi_t \to 0$. Yet suppose the opposite, that is, that $\pi_t \to 1$. If such were the case, the population would consist of only testing cooperators which, from (i) in Section 3, cannot hold. As K assumes values that increasingly move it away from being close to $R - P$ toward close to zero, the associated values of π_t must become *smaller*. To see the reason for this result, suppose that an equilibrium holds at $R - (K_0/\pi_{t_0}) = P$ and consider the opposite, that is, as K declines from K_0 to K_1, π_t *increases* from π_{t_0} to π_{t_1}. But then $(K_1/\pi_{t_1}) < (K_0/\pi_{t_0})$, rendering it impossible to restore equilibrium at $R - (K_0/\pi_{t_0}) = P$. As long as R and P are given, observing the equilibrium requires that π_t and K move in tandem.

To complete the characterization of the equilibrium we note that in order for there to be no non-testing cooperators in the population, it has to be the case that if a non-testing cooperator were to enter the population, he will receive a lower payoff than that received by the testing cooperators and the defectors, that is, $\pi_t R + (1 - \pi_t)S < P$. Substituting $\pi_t = K/(R - P)$ and rearranging terms we get

$$K < \frac{(P - S)(R - P)}{R - S} = \frac{P - S}{R - S}\bar{K} < \bar{K}$$

since $(P - S) < (R - S)$. Hence, exclusion of non-testing cooperators requires that

$$K < \frac{P - S}{R - S}\bar{K} \equiv \bar{\bar{K}}.$$

5. The relationship between the equilibrium fraction of cooperators in a population and a population's level of wealth

Suppose we compare two populations that are equal in all respects except that one population is uniformly wealthier than the other population. By "uniformly" we mean that there are no distributional differences in the payoffs to strategies; the only difference between the two populations is that in one population the payoffs are uniformly higher than in the other population, say by a factor of $\mu > 1$. Holding K constant, $\pi_t^w = K/\mu(R - P)$ of the wealthier population is smaller than $\pi_t = K/(R - P)$ of the less wealthy population: the equilibrium fraction of cooperators in a wealthy population is smaller than the equilibrium fraction of cooperators in a (uniformly) less wealthy population.[1]

To appreciate the nature of this outcome consider the case of $\pi_t = K/\mu(R - P)$ where $\mu \to \infty$. It follows that $\pi_t \to 0$. The implication of a rising μ is that the absolute difference

[1] To rule out the possibility that, in spite of the payoffs to every cooperator and to every defector being higher in the wealthier population, the payoff per capita (and, since population size is held constant, total wealth) will be lower in the wealthier population, the sufficient condition that $\mu > \underline{\mu} \equiv \pi_t/\pi_t^w$ can be added. This condition arises from the requirement that the per capita payoff in the wealthier population will be higher than the per capita payoff in the less wealthy population:

$$\pi_t^w\left(\mu R - \frac{K}{\pi_t^w}\right) + (1 - \pi_t^w)\mu P > \pi_t\left(R - \frac{K}{\pi_t}\right) + (1 - \pi_t)P.$$

between the payoffs R and P becomes increasingly larger. With K held constant, if π_t were, alternatively, to *rise*, the expected payoff of testing cooperators will increasingly distance itself from the expected payoff of defectors (who, it will be recalled, play only with defectors) and equilibrium will not be restored.

Two comments regarding recognition costs are in order. First, for the equilibrium to hold, K can take a wider range of values than before since the constraint pertaining to K, which is now $K < \mu\bar{\bar{K}}$, is less stringent. Second, the inverse relationship between the equilibrium fraction of cooperators and the level of wealth holds even when K increases with wealth, provided that the increase is less than μ. An increase in wealth is due to and entails a first order increase in the payoffs from trade and exchange but, at most, a second order increase in the cost of conducting trade. Indeed, in a population whose level of wealth is higher, the recognition cost could be lower (for example, a computerized credit inquiry could replace a lengthy interview). If $K = K(\mu)$ and $K'(\mu) < 0$, then $\pi_t^w = K(\mu)/\mu(R - P)$ and

$$\frac{\partial \pi_t^w}{\partial \mu} = -\frac{K(\mu)}{\mu^2(R-P)} + \frac{K'(\mu)}{\mu(R-P)} < -\frac{K(\mu)}{\mu^2(R-P)};$$

the adverse effect of a higher level of wealth on the equilibrium fraction of cooperators is stronger.

6. Robustness of the cooperation–wealth relationship when the testing cooperators are somewhat adventurous

Suppose that a testing cooperator acts in the following manner: with probability $0 < q \leq 1$ he administers the type-recognition test. With probability $1 - q$ he does not resort to the test and plays with whomever he happens to be paired with. (We know that q cannot be equal to zero because then we will have only defectors and non-testing cooperators which, from (ii) in Section 3, cannot be the case in equilibrium.) We seek to find out whether the result of Section 5 holds in this setting too.

The expected payoff of an adventurous testing cooperator is

$$V_t^a = \frac{(1-\pi_d)R + (1-q)\pi_d S - qK}{1 - q\pi_d}. \tag{5}$$

The proof is as follows: when testing occurs with probability q, a match will confer a payoff either when the test is applied (at a cost K) and the partner in the match is found to be a cooperator, a case in which the play yields $q[-K + (1-\pi_d)R]$, or when the test is not applied, a case in which the payoff received is $(1-q)[(1-\pi_d)R + \pi_d S]$. In the event that the test is applied and the partner to the match is found not to be a cooperator, which occurs with probability $q\pi_d$, no payoff is received and the entire procedure is repeated thereby yielding V_t^a. Thus,

$$\begin{aligned} V_t^a &= q[-K + (1-\pi_d)R] + (1-q)[(1-\pi_d)R + \pi_d S] + q\pi_d V_t^a \\ &= \frac{(1-\pi_d)R + (1-q)\pi_d S - qK}{1 - q\pi_d}. \end{aligned} \qquad \square$$

Since the combined population share of testing cooperators who happen not to administer the test and of defectors is $1 - q\pi_{\mathrm{t}}$, the expected payoff of a defector is

$$V_{\mathrm{d}} = \frac{1 - q\pi_{\mathrm{t}} - \pi_{\mathrm{d}}}{1 - q\pi_{\mathrm{t}}} T + \frac{\pi_{\mathrm{d}}}{1 - q\pi_{\mathrm{t}}} P$$

or

$$V_{\mathrm{d}} = T - \frac{\pi_{\mathrm{d}}}{1 - q\pi_{\mathrm{t}}}(T - P). \tag{6}$$

In equilibrium, adventurous cooperators receive the same expected payoff as defectors. Thus, from (5) and (6),

$$\frac{(1 - \pi_{\mathrm{d}})R + (1 - q)\pi_{\mathrm{d}} S - qK}{1 - q\pi_{\mathrm{d}}} = T - \frac{\pi_{\mathrm{d}}}{1 - q\pi_{\mathrm{t}}}(T - P).$$

Of course, $q\pi_{\mathrm{t}} + (1 - q)\pi_{\mathrm{t}} = \pi_{\mathrm{t}}$ and hence $\pi_{\mathrm{t}} + \pi_{\mathrm{d}} = 1$. We therefore have that

$$\frac{\pi_{\mathrm{t}} R + (1 - q)(1 - \pi_{\mathrm{t}}) S - qK}{1 - q(1 - \pi_{\mathrm{t}})} = T - \frac{1 - \pi_{\mathrm{t}}}{1 - q\pi_{\mathrm{t}}}(T - P). \tag{7}$$

Evaluating this last equality at $q = 1$ yields

$$\frac{\pi_{\mathrm{t}} R - K}{\pi_{\mathrm{t}}} = T - (T - P)$$

or

$$\pi_{\mathrm{t}} = \frac{K}{R - P}.$$

By continuity this last equality holds for values of q in (7) that are in the small neighborhood of 1. Hence, the cooperation–wealth relationship alluded to in Section 5 holds also when testing cooperators apply the test with a probability that is less than, but close to, one.

7. Conclusion

We calculate the equilibrium fraction of cooperators in a population in which payoffs are received upon playing a two-person single-shot prisoner's dilemma game; individuals who are hardwired as cooperators or as defectors are paired randomly; cooperators check, at a cost, the type of individual with whom they are paired prior to executing a game and play only with cooperators; and defectors play with whomever they happen to be paired. Measuring the wealth of a population by the level of the payoffs in the prisoner's dilemma game, we show that the wealthier the population, the lower the equilibrium fraction of cooperators.

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